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Technical Comments

Comment on "Aircraft with Single-Axis Aerodynamically Deployed Wings"

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IN Ref. 1, the authors raise the question: "Is there an axis fixed in both [wing] B and [fuselage] A, such that, if B is rotated around the indicated axis relative to A an amount θ, starting in the [stored] configuration . . . , then B ends up having the [deployed] configuration . . . ?" They say they were able to show that such an axis exists.

This question is but a special case of one that Leonard Euler raised and solved long ago.[2] A famous theorem, due to Euler, states that any two orientations can be bridged by a single rotation around a fixed axis.

The angle of this rotation may be extracted from the orthogonal matrix R describing the relative orientation by

$$\cos\theta = \tfrac{1}{2}(R_{11} + R_{22} + R_{33} - 1) \tag{1}$$

The components of a unit vector $\boldsymbol{e}$ along the rotation axis are given by

$$e_1 = \frac{R_{32} - R_{23}}{2\sin\theta} \tag{2}$$

$$e_2 = \frac{R_{13} - R_{31}}{2\sin\theta} \tag{3}$$

$$e_3 = \frac{R_{21} - R_{12}}{2\sin\theta} \tag{4}$$

Equation (1) determines two angles. In general, one is smaller than π and the other larger than π. Whichever angle is used, the required rotation is in the right screw sense around the vector $\boldsymbol{e}$. See Ref. 3 for further details.

Euler's theorem states that the axis of rotation is unique. How is it, then, that in Ref. 1, two axes are found? This is because Ref. 1 addresses a rectangular wing and does not distinguish between the root and the tip. When a more general wing is admitted and the root and tip are predefined, only one axis exists for the deployment with one angle smaller than π (120 deg in the case studied in the reference), and one larger than π (240 deg for the case in the reference). Case DL has the wing stored with the tip forward, and case UL with the tip aft.

Both cases assume that the wing is stored with the top surface on the outside. If storage with the bottom surface on the outside was considered, two new deployment axes would become available. If the stored position were rotated to lie partway down the fuselage and/or the deployed position were allowed sweep or dihedral, further variations in deployment geometry would result with the angle no longer being 120 deg. Also, taper of the fuselage or of the wing, with the wing still stored hugging the fuselage, would cause the deployment angle to deviate from 120 deg.

In conclusion, the designer is free to select the stored orientation and the deployed orientation to advantage. Once this is done, Eqs. (1–4) determine the axis and angle of deployment. The analysis of Djerassi and Kotzev can then be applied.

References

[1]Djerassi, S., and Kotzev, S., "Aircraft with Single-Axis Aerodynamically Deployed Wings," *Journal of Aircraft*, Vol. 32, No. 2, 1995, pp. 343–348.

[2]Euler, L., *Novi Commentarii Academiae Petropolitanae*, Vol. 15, 1770, pp. 13–15, 75–106.

[3]Katz, A., *Rigid Vehicle Dynamics*, Univ. of Alabama Academic Publishing Service, Tuscaloosa, AL, 1991, 1995.

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Reply by the Author to A. Katz

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THE author of Ref. 1 draws attention to a question raised in Ref. 2, and to its relation to a theorem by Euler. Moreover, he concludes that Eqs. (1–5) in Ref. 2 were obtained under the assumption that the wing of the aircraft discussed in Ref. 2 is rectangular. It will be shown here that, in fact, such an assumption has not been made.

To this end, suppose wing B undergoes a simple rotation in fuselage A (i.e., a rotation about an axis L fixed in both A and B) of the aircraft described in Fig. 1 of Ref. 2. Let $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ (i = 1, 2, 3) be two sets of dextral, mutually perpendicular unit vectors fixed in A and in B, respectively, as shown in Fig. 1. Choosing $\boldsymbol{a}_i = \boldsymbol{b}_i$ (i = 1, 2, 3) when B is deployed, note that, from an engineering point of view, the deployed configuration of B in A is unique, and that there are four admissible stored

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